Physics II
ISI B.Math
Final Exam : November 15, 2017

Total Marks: 50
Time: 3 hours
Answer all questions

1. **(Marks: 3 + 4 + 3 = 10)**

Two identical bodies, each characterized by a heat capacity C at constant pressure which is independent of temperature, are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially , their temperatures are T_1 and T_2 respectively:finally, as a result of the operation of the heat engine, the bodies will attain a common final temperature T_f

(a) What is the total amount of work W done by the engine ? Express the answer in terms of C, T_1, T_2 and T_f

(b) Use arguments based on entropy considerations to find an inequality relating T_f and T_1 and T_2

(c) For given initial temperatures T_1 and T_2, what is the maximum amount of work obtainable from the engine ?

2. **(Marks: 3 + 3 + 4 = 10)**

Einstein introduced a simplified model of solids where a solid is a collection of independent quantum harmonic oscillators of the same angular frequency ω. The energy accessible to each oscillator is given by $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2, 3 \cdots$. One mole of the solid therefore contains $3N_a$ independent one-dimensional harmonic oscillators where N_a is Avogadro's number.

(a) Find the partition function for this system for one mole of the solid.

(b) Find the average energy.

(c) Show that the molar specific heat of the solid based on Einstein's model is given by:

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\frac{\theta_E}{T}}}{(e^{\frac{\theta_E}{T}} - 1)^2}$$

where $R = N_a k$ and $\frac{\hbar\omega}{kT} = \frac{\theta_E}{T}$.
Make a rough plot of the specific heat as a function of temperature.Is the low temperature behaviour consistent with the third law of thermodynamics?

3. **(Marks: 3 + 4 + 3 = 10)**

Consider a system of N atoms with spin such that when placed in a magnetic field, the spin (or intrinsic magnetic moment) can align itself either parallel or antiparallel to the magnetic field. When

the spin is aligned along the magnetic field the energy of the atom is $-\epsilon$ and when it is aligned opposite to the magnetic field the energy of the atom is ϵ.

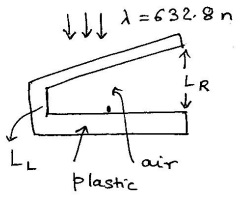
i) Find the number of microstates Ω corresponding to a state of total energy E

ii) Find the entropy $S(N, E)$ of the system using the result in i). Since N is large, feel free to use Stirling's approximation for large N. Make a rough plot of entropy S as a function of energy E.

$$\log N! \simeq N \log N - N$$

iii) Find the temperature $T(E)$ of the system. Can the temperature be negative ? Explain.

4. **(Marks : 5 + 5 = 10)**



(a) Transparent plastic material with the shape given in the figure encloses a thin wedge of air inside as shown . A broad beam of red light with wavelength $\lambda = 632.8$ nm is directed downward through the top of the plastic. Some of the light is reflected back up from the top and bottom surfaces of the wedge, which acts as a thin film of air with a thickness that varies uniformly and gradually from L_L at the left to L_R at the right. An observer looking down at the wedge sees an interference pattern consisting of six dark fringes and five bright red fringes along the wedge. What is the change in thickness $\Delta L = (L_R - L_L)$ along the wedge?

(b) A Newton's ring arrangement of a plano convex lens placed on a glass plate is illuminated by monochromatic light of wavelength $\lambda = 6.4 \times 10^{-5}$ cm . The radius of the convex surface is 100 cm. Assume the point of contact is perfect. The refractive index of the glass is 1.5

(i) Will the central spot of the ring system be dark or bright ? Explain.

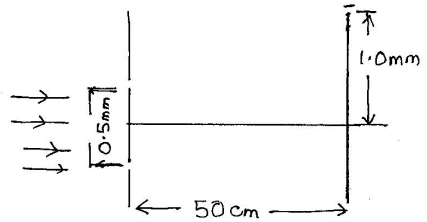
(ii) The lens is now raised vertically above the plate by a distance 1.6×10^{-5} cm. Will the central spot be dark or bright ? Explain.

(iii) The lens is then put back on the glass plate such that the point of contact is perfect. The whole arrangement is now immersed in water of refractive index 1.33. Will the central spot be dark or bright ? Explain.

(iv) The above arrangement is immersed in oil of refractive index 1.7 instead of water. What will be the nature of the central spot ?

(v) If the configuration considered in (i) is illuminated with white light instead of monochromatic light, what will be the nature of the central spot ?

5. **(Marks : 3 + 3 + 2 + 2 = 10)**



(a) White coherent light(400nm -700nm) is sent through the slits of a Young's double slit experiment. The separation between the slits is 0.5 mm and the screen is 50 cm away from the slits. There is a hole in the screen at a point 1.0 mm away(along the width of the fringes) from the central line. Which wavelength(s) will be absent in the light coming through the hole ?

(b) Consider a double slit arrangement with each slit of width b and the separation between slits equal to d. Derive an expression for the number of interference maxima occurring under the central diffraction minimum of the double slit pattern in terms of d and b.

(c) Explain why in a grating made out of alternate transparent and opaque strips of equal width all the even order interference maxima (excluding $m = 0$) are absent.

(d)What is the state of polarization of a wave whose x and y components of the electric field are given by $E_x = E_0 \cos(\omega t + kz)$ and $E_y = \frac{E_0}{\sqrt{2}} \cos(\omega t + kz + \pi)$, where the symbols have their usual meanings?